# MODE PERIOD TIME HISTORY OF AN EMBANKMENT COMPUTED FROM VERSAT-2D by Dr. G. WU

In a VERSAT-2D time history anlaysis, the 1st and 5th mode frequencies of the finite element model as a whole are computed by solving the eigenvalue equation:

[1] $$[K] - \omega_i^2 [M] = 0$$

Where [M] and [K] are the mass matrix and stiffness matrix of the model, respectively; and $\omega_i$ is the $i^{th}$ radian frequency. In a nonlinear dynamic analysis, the stiffness of the model varies with time as shear moduli of soil elements within the model reduce when the level of ground shaking increases; therefore, time histories of the 1st and 5th mode frequencies ($\omega_1$ and $\omega_5$) of the finite element model are computed and saved in an output file, i.e., the *.O21 file. The 1st mode period is then $T_1 = 2\pi/\omega_1$, and the 5th mode period is $T_5 = 2\pi/\omega_5$.

### 1D Soil Column

A 1D column model of the foundation soil is shown in part of Figure 1. The 1D model consists of 20 soil elements, constructed using the method described in Section 4.7 of VERSAT-2D User Manual on "*Dynamic Analysis of One Dimensional Soil Column*". Dynamic analysis of 1D column is quick and only takes a few minutes to complete one run using the 40 sec long input motion shown in Figure 2.

The original Taiwan Chi-Chi earthquake acceleration recorded at station TCU071 W has a duration of 90 sec and a PGA of 0.567 g. The first 20 sec and the last 30 sec of this record are truncanted; the remaining 40 sec long portion is used in current dynamic analyses, as shown in Figure 2. This acceleration record is further linearly scaled down to PGAs of 0.12 g, 0.27 g and 0.48 g after applying scale factors of 0.21, 0.48 and 0.84, respectively, to create three levels of ground shaking.

Results of 1st and 5th mode periods for the 1D column are shown in Figure 3 and in Table 1. It is noted that only the first 20 sec of the response acceleration at the top of the 1D column is shown in order to better present the time history trace for the portion of strong shaking. The analytical solution for 1st mode period ($T_1$) of the 1D column is:

[2] $$T_1 = 4H / V_s$$

For a 20 m high soil column with an assumed shear wave velocity of $V_s$=300 m/s, the 1st mode period is calculated to be $T_1$ = 0.27 sec, which is identical to the value computed by VERSAT-2D (at t=0 sec in Figure 3).

Table 1 The range of 1st and 5th mode periods for the 1D column under three levels of shaking

| Shaking level | Input motion PGA (g) | 1st Mode Period (sec) | 5th Mode Period (sec) |
|---|---|---|---|
| 1 | 0.12 | 0.27 – 0.35 | 0.05 – 0.05 |
| 2 | 0.27 | 0.27 – 0.67 | 0.05 – 0.06 |
| 3 | 0.48 | 0.27 – 1.83 | 0.05 – 0.14 |

### 2D Embankement

The results of 1st ad 5th mode periods for an example 2D embankement with a crest width of 20 m and a side slope of 2H:1V are shown in Figure 4 and in Table 2. Again, only the first 20 sec of the response acceleration (this time at the embankment crest) is shown. The elastic 1st mode period computed by VERSAT-2D is 0.41 sec (at t = 0 sec in Figure 4) which lies in the middel of the two first mode periods ($T_1$), one for the 1D foundation column (H=20 m) with $T_1$ = 0.27 sec and the other one for the 1D column consisting of the foundation and the embankment (H=40 m) with $T_1$ = 0.53 sec.

The following observations are made from results of dynamic analyses of the 2D example embankment:

- When subject to low level of ground shaking with a PGA of 0.12g, fundamental period ($T_1$ ) of the 2D embankment can increase by 50% from its elastic value of 0.41 sec to 0.62 sec due to soil stiffness reduction. The $T_1$ range widens to 0.41 sec – 0.80 sec subject to a moderate level of shaking with a PGA of 0.27 g.
- When subject to a high level of shaking with a PGA of 0.48 g, the fundamental periods ($T_1$) of the 2D embankment can range from 0.41 sec – 1.75 sec, a much wider range than for the low to moderate shaking levels.
- The 5th mode periods of the 2D embankemnt are much shorter than the 1st mode, with a range in the order of 0.25 sec under low level of shaking; the range widens to 0.23 sec – 0.66 sec under a high level of shaking. However, contribution to response from high frequency modes including the 5th mode can't be ignored in a dynamic analysis. For example a 5th mode period of 0.25 sec would fall in the range of the predominant periods of some input ground motions from shallow crustal earthquakes.

Table 1 The range of 1st and 5th mode periods for the 2D embankement under three levels of shaking

| Shaking level | Input motion PGA (g) | 1st Mode Period (sec) | 5th Mode Period (sec) |
|---|---|---|---|
| 1 | 0.12 | 0.41 – 0.62 | 0.23 – 0.29 |
| 2 | 0.27 | 0.41 – 0.80 | 0.23 – 0.37 |
| 3 | 0.48 | 0.41 – 1.75 | 0.23 – 0.66 |

Figure 1 VERSAT-2D Finite Element Models for Dynamic Analyses of An Example Embankment

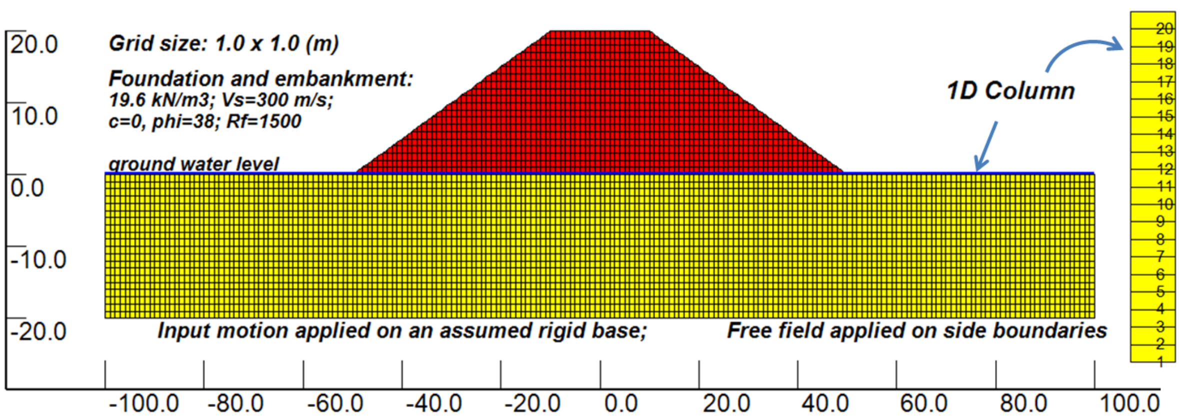


Figure 2 Unscaled Input Acceleration Time History from the Chi Chi Earthquake (PGA of 0.567g)

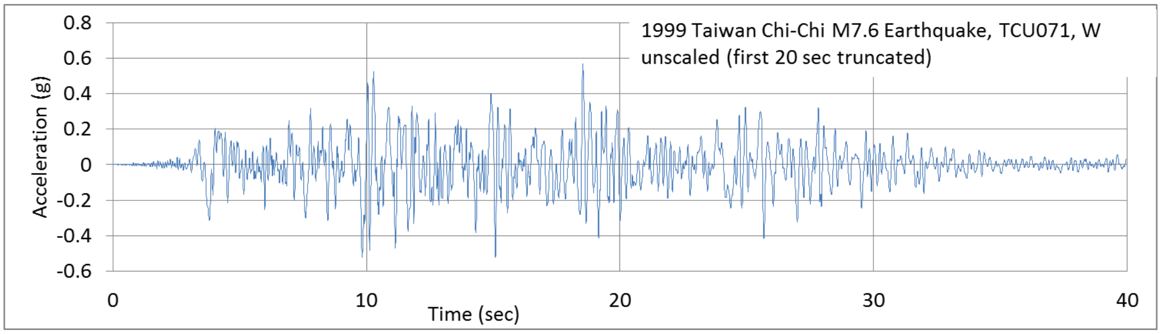

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Figure 3 Computed Time-History of 1st and 5th Mode Periods for the 1D Soil Column under Three Levels of Shaking

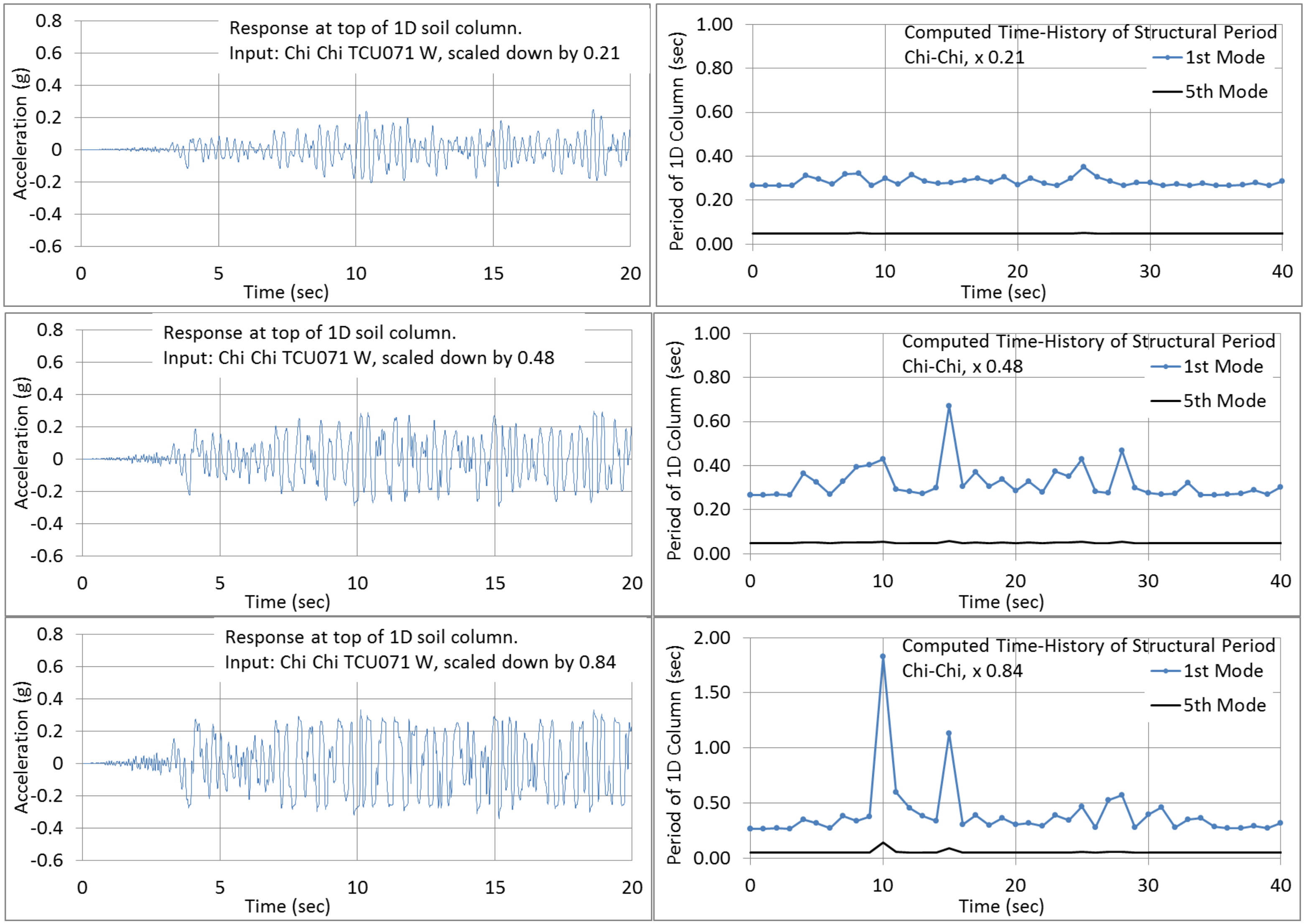

Figure 4 Computed Time-History of 1st and 5th Mode Periods for the 2D Embankment under Three Levels of Shaking